$\int \frac{f'(x)}{\operatorname{sen}^2 f(x)}$ $\int f'(x)\cdot \operatorname{cosec}^2 f(x)\,dx = \int [1+\operatorname{cotg}^2(f(x))] f'(x)\,dx = -\operatorname{cotg}(f(x))+C$

$\int \frac{f'(x)}{1+(f(x))^2}\,dx = \operatorname{arctg} f(x) = -\operatorname{arccotg} f(x)$

4ª EVALUACIÓN

1º B

1º Dada $f(x) = \frac{x^3}{(x-3)^2}$

- Calcula sus asíntotas
- Ecuación de la tg. a $f(x)$ en $x_0 = 2$ 1'5

2º Estudia y representa $y = x e^{-x^2}$
Intégrala 2'5

3º Calcula $f(x)$ sabiendo $f''(x) = 2x - 1$ y $f'(0) = 1$ y $f(-1) = 2$
0'75

4º Esquema y area determinada $y = \frac{5}{x^2+1}$ e $y = 1$
1'25

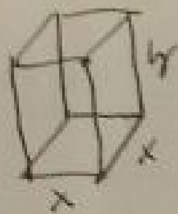
5º Integra:

a) $\int \frac{x-3}{x^2+9}\,dx$ b) $\int_1^2 \frac{3 - x^2 + x^4}{x^3}\,dx$ c) $\int \frac{\cos x}{5+\operatorname{sen} x}\,dx$ d) $\int \cos^4 x \operatorname{sen} x\,dx$

e) $\int 3^{\operatorname{tg} x} \frac{1}{\cos^2 x}\,dx$ 1'5 ptos

6º Se dispone de $1200\,m^2$ de chapa para construir un deposito en forma de prisma recto de base cuadrada sin tapa superior. Calcula sus dimensiones, para que el volumen sea maximo. Calcula el volumen.

1'5




7º Area determinada $\begin{cases} y = x^2 - 2x + 2 \\ y = -8x + 6 \\ OX \\ x = 1 \end{cases}$

1 pto

P. Inte

1º) $\int$

① a) $D = \forall \mathbb{R} - \{3\}$ AV: $\lim_{x\to 3^-} = \frac{+}{0^+} = \boxed{\infty}$; $\lim_{x\to 3^+} = \frac{+}{0^+} = \boxed{\infty}$ AH: $\lim_{x\to -\infty} = \frac{-\infty}{\infty} = \boxed{-\infty}$; $\lim_{x\to\infty} = \frac{\infty}{\infty} = \boxed{\infty}$

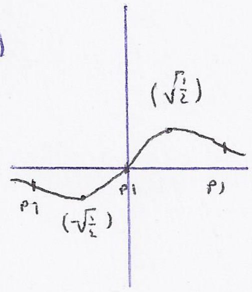
NO AH — gr num > gr denominador

AO: $m = \lim_{x\to\infty} \frac{x^3}{x(x-3)^2} = 1$ $n = \lim_{x\to\infty}\left(\frac{x^3}{(x-3)^2} - x\right) = \lim_{x\to\infty} \frac{x^3 - x^3 - 9x + 6x^2}{(x-3)^2} = 6$; $\boxed{y = x+6}$

b) $P(2, 8)$: $f'(x) = \frac{3x^2(x-3)^{\cancel{2}} - 2x^3\cancel{(x-3)}}{(x-3)^{\cancel{4}\,3}}$; $f'(2) = \frac{-12-8}{-1} = 20$; $\boxed{y - 8 = 20(x-2)}$

② $y = \frac{x}{e^{x^2}}$; $D = \forall \mathbb{R}$; Simetría respecto a O (0,0). AH: $\lim_{x\to\pm\infty} = \frac{\pm\infty}{e^{\infty}} = \pm\frac{\infty}{\infty} = \begin{cases} 0^+ \\ 0^- \end{cases}$

NO AV — crece más rápido $e^x$




$y' = \frac{\cancel{e^{x^2}} - x\cancel{e^{x^2}}\,2x}{(e^{x^2})^{\cancel{2}}} = \frac{1-2x^2}{e^{x^2}} = 0$; $x^2 = \frac{1}{2}$; $x = \pm\sqrt{\frac{1}{2}} \rightarrow (-\infty, -\sqrt{\frac{1}{2}})(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{2}})(\sqrt{\frac{1}{2}}, \infty)$

min rel $(-\sqrt{\frac{1}{2}}, -\frac{1}{2\sqrt{e}})$ max rel $(\sqrt{\frac{1}{2}}, \frac{1}{2\sqrt{e}})$

$y'' = \frac{-4x\cancel{e^{x^2}} - 2x\cancel{e^{x^2}}(1-2x^2)}{(e^{x^2})^{\cancel{2}}} = \frac{-4x - 2x + 4x^3}{e^{x^2}} = 0$; $x(4 - 6x^2) = 0$; $x = 0$; $x = \pm\sqrt{\frac{2}{3}}$; $(-\infty, \sqrt{\frac{2}{3}})(\sqrt{\frac{2}{3}}, 0)(0, \sqrt{\frac{2}{3}})$

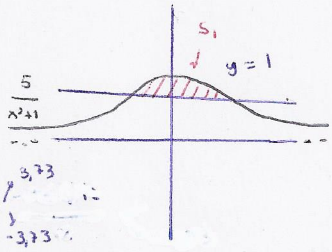
$(0,0)$ $(\pm\sqrt{\frac{2}{3}}, \pm\frac{\sqrt{\frac{2}{3}}}{\sqrt[3]{e^2}})$ $(\sqrt{\frac{2}{3}}, \infty)$

$\int \frac{x}{e^{x^2}}\,dx = -\frac{1}{2}\int 2x\,e^{-x^2} = -\frac{1}{2}e^{-x^2} + C_1$

③ $f'(x) = \int 2x - 1\,dx = x^2 - x + C_1$; $C_1 + 0 - 0 = 1$; $f(x) = \int x^2 - x + 1\,dx = \frac{x^3}{3} - \frac{x^2}{2} + x + C_1$; $-\frac{1}{3} - \frac{1}{2} - 1 + C_1 = 2$;

$C_1 = \frac{23}{6}$; $\boxed{f(x) = \frac{x^3}{3} - \frac{x^2}{2} + x + \frac{23}{6}}$

④ $D = \forall \mathbb{R}$; Simetría OX AH: $\lim_{x\to\pm\infty} = \frac{5}{\infty} = 0^+$; $y' = -\frac{10x}{(x^2+1)^2} = 0$; $x = 0$ $(0,5)$




$y'' = \frac{-10(x^2+1)^{\cancel{2}} + 20x\,\cancel{(x^2+1)}\,2x}{(x^2+1)^{\cancel{4}\,3}} = \frac{-10x^2 - 10 + 40x}{(x^2+1)^3}$; $x^2 - 4x + 1 = 0$; $x = \frac{+4 \pm\sqrt{16-4}}{2} = \begin{cases} 3{,}73 \\ -3{,}73 \end{cases}$

$\frac{5}{x^2+1} = 1$; $5 = x^2 + 1$, $x = \pm 2$; $S_1 = \int_0^2 \frac{5}{x^2+1} - 1\,dx = \int_0^2 \frac{5 - x^2 - 1}{x^2+1}dx = \int_0^2 \frac{4 - x^2}{x^2+1} = \int_0^2 \frac{4}{x^2+1}dx - \int_0^2 \frac{x^2}{x^2+1} - 1 + 1\,dx =$

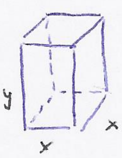
$= \left[4\,\text{arctg}\,x + \text{arctg}\,x + \cancel{x}\right]_0^2 = \left[5\,\text{arctg}\,x - x\right]_0^2 = 5\,\text{arctg}\,2 - 2\,u^2$; $S_T = 2S_1 =$ $\frac{x^2 - x^2 - 1}{x^2+1} + 1$

$= 2(5\,\text{arctg}\,2 - 2) = \boxed{5{,}54\,u^2}$ ⚠ Radianes

$\boxed{(\frac{21}{8} - L2)\,u^2}$

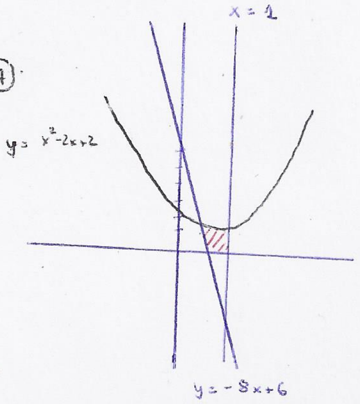
⑤ $\int \frac{x-3}{x^2+9}dx = \frac{1}{2}\int \frac{2x}{x^2+9}dx - \int \frac{\overbrace{3/9}^{1/3}}{\frac{9}{9} + \frac{x^2}{9}} = \boxed{\frac{1}{2}L(x^2+9) - \frac{1}{3}\text{arctg}\,x + C_1}$ $\int_1^2 \frac{3 - x^2 + x^4}{x^3}dx = \left[\frac{-3}{2x^2} - Lx + \frac{x^2}{2}\right]_1^2 = \frac{13}{8} - L2 - (-1)$

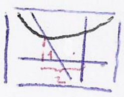
$\int \frac{\cos x}{5 + \text{sen}\,x}dx = \boxed{L(5 + \text{sen}\,x) + C_1}$ $\int \cos^4 x\,\text{sen}\,x\,dx = \boxed{-\frac{\cos^5 x}{5} + C_1}$ $\int 3^{\text{tg}\,x}\frac{1}{\cos^2 x}dx = \boxed{\frac{3^{\text{tg}\,x}}{L3} + C_1}$

⑥ 


$\left.\begin{array}{l} x^2 + 4xy = 1200 \\ x^2 y = V \end{array}\right\}$ $y = \frac{1200 - x^2}{4x}$; $V = x^{\cancel{2}}\frac{1200 - x}{4\cancel{x}} = \frac{1}{4}x(1200 - x)$ $V' = \frac{1}{4}\left[\overbrace{1200 - x^2 - 2x^2}^{1200 - 3x^2}\right] =$

$= \frac{3}{4}(400 - x^2) = 0$; $x^2 = 400$; $\boxed{x = 20\,m \quad y = 10\,m}$ $V = 400 \cdot 10 = \boxed{4000\,m^3}$

⑦ 

$-8x + 6 = x^2 - 2x + 2$; $x^2 + 6x - 4 = 0$; $x = \frac{-6 \pm\sqrt{36+16}}{2} = \frac{-6 \pm\sqrt{52}}{2} = \underbrace{-3 \pm \frac{\sqrt{52}}{2}}_{-3 + \sqrt{13} = 0{,}6}$

$0 = -8x + 6$; $x = \frac{6}{8} = \frac{3}{4}$; $S_1 = \frac{1}{2}\left(\frac{3}{4} + 3 - \sqrt{13}\right) 1 =$ Base Altura

$= \frac{15 - 4\sqrt{13}}{8}\,u^2 = 0{,}07\,u^2$

$S_2 = \int_{0,6}^{1} x^2 - 2x + 2 = \left[\frac{x^3}{3} - x^2 + 2x\right]_{0,6}^{1} = \frac{1}{3} - 1 + 2 - (0{,}912) = \frac{4}{3} - 0{,}912 =$

$= 0{,}42\,u^2$;

$S_T = S_2 - S_1 = 0{,}42 - 0{,}07 = \boxed{0{,}35\,u^2}$