8-11-13 Corrección del examen de la 1ª evaluación (8,5)

① Calcular en función de $\log 2$ el valor de $A = \dfrac{40\sqrt[25]{320}}{0{,}003}$;

$$A = \frac{(2^2\cdot 10)^2\cdot(2^5\cdot 10)^{2/5}}{2^3\cdot 10^{-3}} = \frac{2^4\cdot 10^2\cdot 2\cdot 10^{\frac{1}{5}}}{2^3\cdot 10^{-3}} = \frac{2^5\cdot 10^{\frac{11}{5}}}{2^3\cdot 10^{-3}} = 2^2\cdot 10^{\frac{26}{5}};$$

$$\log A = 2\log 2 + \frac{26}{5}\log 10 = \boxed{2\log 2 + \frac{26}{5}}$$

② Calcular el valor de: $\log_2 8 + \log_8 \sqrt[4]{2} - \log_5\sqrt{125} + \log 0{,}001$

$$3 + \frac{1}{12} - \frac{3}{2} - 3 = \boxed{-\frac{17}{12}}$$

③ a) $\log x + \log(x+3) = 2\log(x+1) = x(x+3) = (x+1)^2;\ x^2+3x = x^2+1+2x;\ \boxed{x=1}$

b) $4\log\left(\frac{x}{5}\right) + \log(x+3) = 2\log(x+1) = \left(\frac{x}{5}\right)^4\cdot\frac{625}{4} = x^2;\ \frac{x^4}{4} = x^2;$

$x^4 = 4x^2;\ x = \pm 2;\ 4\log\frac{-2}{5} < 0 \Rightarrow$ no vale; $\boxed{x=2}$ $x=0$ no vale.

c) $\left.\begin{array}{l} x+y=29 \\ \log x + \log y = 2 \end{array}\right\}$ $\left.\begin{array}{l} x+y=29 \\ xy = 100 \end{array}\right\}$ $x = 29-y;\ (29-y)y = 100;\ y^2-29y+100=0;$

$y = \dfrac{29\pm\sqrt{841-400}}{2} = \dfrac{29\pm 21}{2}$ $\boxed{y=25}$ $\boxed{y=4}$

d) $\left.\begin{array}{l} -1 \le \dfrac{x-3}{x+2} \\ \dfrac{x-3}{x+2} \le 1 \end{array}\right\}$ $\dfrac{-x-2}{x+2} \le \dfrac{x-3}{x+2};\ \dfrac{2x-1}{x+2} \ge 0;$

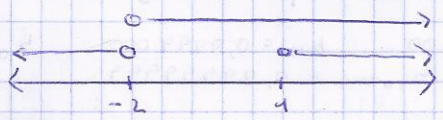
	$(-\infty,-2)$	$(-2,\frac{1}{2})$	$(\frac{1}{2},\infty)$
$2x-1$	−	−	+
$x+2$	−	+	+
	+	−	+

$x \in (-\infty,-2)\cup[\frac{1}{2},\infty)$

$\dfrac{x-3}{x+2} \le \dfrac{x+2}{x+2};\ \dfrac{x+2-x+3}{x+2} \ge 0;\ \dfrac{5}{x+2} \ge 0;$

	$(-\infty,-2)$	$(-2,\infty)$
$x+2$	−	+

$x \in (-2,+\infty)$



$\boxed{x \in [1,\infty)}$

e) $\dfrac{(x^2+9)(x^2+4x+3)}{(2-x)(x^2-6x+8)} \ge 0;\ \dfrac{(x^2+9)(x+1)(x+3)}{(2-x)(x-4)(x-2)} \ge 0;$

	$(-\infty,-3)$	$(-3,-1)$	$(-1,2)$	$(2,4)$	$(4,\infty)$
$(x+1)$	−	−	+	+	+
$(x+3)$	−	+	+	+	+
$(2-x)$	+	+	+	−	−
$(x-4)$	−	−	−	−	+
$(x-2)$	−	−	−	+	+
	+	−	+	+	−

$x \in (-\infty,-3]\cup[-1,2)\cup(2,4)$

$x^3-x^2-16x+16>0$
$(x-2)(x+3)(x-1)\le 0$ $\Big\}$ $x^2(x-1)-16(x-1)>0;\ (x^2-16)(x-1)>0;$

$(x+4)(x-4)(x-1)>0;$

	$(-\infty,-4)$	$(-4,1)$	$(1,4)$	$(4,\infty)$
$(x+4)$	−	+	+	+
$(x-4)$	−	−	−	+
$(x-1)$	−	−	+	+
	−	+	−	+

$x \in (-4,1)\cup(4,\infty)$

	$(-\infty,-3)$	$(-3,1)$	$(1,2)$	$(2,\infty)$
$(x-2)$	−	−	−	+
$(x+3)$	−	+	+	+
$(x-1)$	−	−	+	+
	−	+	−	+

$x \in (-\infty,-3]\cup[1,2]$

$\boxed{x \in (-4,-3]}$